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Final Exam MTH 512 , Fall 2018

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$\frac{82}{85}$

QUESTION 1. Let A be a 7×7 matrix such that $m_A(x) = (x-5)^2(x-2)^3$, $dim(E_5) \neq 3$, and $dim(E_2) = 1$.

(i) Explain briefly why $dim(E_5) \neq 1$.

Try to construct Jordan Form of A. Suppose that $dim(E_5) = 1$ & $dim(E_2) = 1$

$J_5^{(2)} \oplus J_2^{(3)}$ each J contributes 1 to dimension

not possible since A is 7×7 $\Rightarrow dim(E_5) \neq 1$

N/N

(ii) Explain briefly why $dim(E_5) \neq 4$.

N/N

Suppose $dim(E_5) = 4$ & $dim(E_2) = 1$

Try to construct $J_5^{(2)} \oplus J_5^{(1)} \oplus J_5^{(1)} \oplus J_5^{(1)} \oplus J_2^{(3)} \Rightarrow 8 \times 8$ not possible

$\Rightarrow dim(E_5) \neq 4$

(iii) Find the Jordan-form of A.

$$J_5^{(2)} \oplus J_5^{(2)} \oplus J_2^{(3)} \Rightarrow \begin{bmatrix} 5 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

(iv) Find $C_A(x)$.

$= (x-5)^4 (x-2)^3$

N/N

QUESTION 2. Let A be a 5×5 matrix such that $m_A(x) = (x-5)^2(x-2)^3$ ← polynomial of degree 5

(i) Find the Jordan-form of A. A is 5×5 $\Rightarrow C_A(x) = m_A(x)$

Jordan form of A is $J_5^{(2)} \oplus J_2^{(3)}$

$$\begin{bmatrix} 5 & 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

(ii) Find $C_A(x)$.

N/N

$C_A(x) = m_A(x) = (x-5)^2(x-2)^3$

(since A is 5×5 $\Rightarrow C_A(x)$ is polynomial of degree 5)

& $m_A(x) \mid C_A(x)$

QUESTION 3. (i) Let A be a non-zero $n \times n$ nilpotent matrix. Prove that A is never diagnolizable but it is always triangulizable.

$A^K = 0$ for some +ve number K

We know from notes that 0 is the only eigenvalue of A

& $C_A(\alpha) = \alpha^n$

We conclude that $A \neq 0_{1\times 1} \Rightarrow n > 1$ (A non zero)

$m_A(\alpha) \neq \alpha$ (since $A \neq$ 0-matrix)

$\Rightarrow m_A(\alpha) = \alpha^m$ for some $2 \leq m \leq n$

$\Rightarrow A$ is not diagonalizable, since $0 \in \mathbb{R}$ (only eigenvalue) $\Rightarrow$ A is always Triangularizable (by class notes)

(ii) Let A be a non-zero 7×7 nilpotent matrix such that $m_A(x) = x^3$ and $dim(E_0) = 3$. Find all possible Jordan-Forms of A.

1) $J_0^{(3)} \oplus J_0^{(3)} \oplus J_0^{(1)}$

2) $J_0^{(3)} \oplus J_0^{(2)} \oplus J_0^{(2)}$

(iii) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

a. Find $C_A(x)$. $= x^3$

b. Find $m_A(x)$. $= x^3$

for number (d) —(***)

$x = 0$ (eigenvalue)

$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow x_1 = 0,\ x_2 = 0,\ x_3$ free

$\Rightarrow \dim(E_0) = 1$ & $m_A(x) = x^3$

A is similar to Jordan form $J_0^{(3)}$

c. If A is similar to a diagonal matrix D, then find D. If not, explain briefly.

A is not similar to a diagonal matrix since $m_A(x) = x^3$

A $n\times n$ is diagonalizable iff $m_A(x) = (x-a_1)(x-a_2)\cdots(x-a_k)$ where $a_1, \ldots, a_k$ are all distinct eigenvalues of A

d. (short answer, maybe you need to stare really well!) Convince me that A is similar to A^T. $A^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$A^T = J_0^{(3)}$ $\dim(E_0) = 1$ (only one J)

$\Rightarrow C_{A^T}(\alpha) = \alpha^3 = m_{A^T}(\alpha)$

Also A is similar to $J_0^{(3)}$ (see above —(***))

$\Rightarrow$ A is similar to A^T

QUESTION 4. (1) Consider the inner product $< f, k >= \int_{-1}^{1} f(x)k(x)\,dx$ on P_3 (over R). We know $P_3 = span\{1, x, x^2\}$ Find an orthogonal basis for P_3 under the given inner product, say $B = \{h, k, d\}$.

Let $v_1 = 1 \quad v_2 = x \quad v_3 = x^2$

$w_1 = 1 \quad w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1 \qquad \langle v_2, w_1\rangle = \int_{-1}^{1} x \cdot 1\, dx = \frac{x^2}{2}\Big]_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$

$\Rightarrow w_2 = x$

$w_3 = v_3 - \frac{\langle v_3, w_2\rangle}{\langle w_2, w_2\rangle} w_2 - \frac{\langle v_3, w_1\rangle}{\langle w_1, w_1\rangle} w_1 \qquad \langle w_1, w_1\rangle = \int_{-1}^{1} 1\, dx = x\Big]_{-1}^{1} = 1 + 1 = 2$

$\langle w_2, w_2\rangle = \int_{-1}^{1} x^2 dx = \frac{x^3}{3}\Big]_{-1}^{1} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$\langle v_3, w_2\rangle = \int_{-1}^{1} x^2 \cdot x\, dx = \frac{x^4}{4}\Big]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0$

$\langle v_3, w_1\rangle = \int_{-1}^{1} x^2 \cdot 1\, dx = \frac{x^3}{3}\Big]_{-1}^{1} = \frac{2}{3}$

$w_3 = x^2 - 0 - \frac{2/3}{2} 1 = x^2 - 1/3 \qquad \Rightarrow B = \{1, x, x^2 - 1/3\}$

← check opp. page

(2) Will B as you constructed in (1) stay an orthogonal basis for P_3 under the FAKE dot product on P_3?explain BRIEFLY

$B' = \{\underset{b_1}{(0,0,1)}, \underset{b_2}{(0,1,0)}, \underset{b_3}{(1, 0, -1/3)}\}$

1/2

$b_1 \cdot b_2 = 0 \qquad b_1 \cdot b_3 = -1/3 \quad \Rightarrow$ not orthogonal under dot product on P_3

(3) Will B as you constructed in (1) stay an orthogonal basis for P_3 under the inner product $< f, k >= \int_0^1 f(x)k(x)\,dx$? Explain BRIEFLY

$\langle 1, x\rangle = \int_0^1 x\, dx = \frac{x^2}{2}\Big]_0^1 = \frac{1}{2} \quad \Rightarrow$ not orthogonal

1/2

QUESTION 5. (a) Let V be an inner product space such that $dim(V) = n$ and $B = \{a_1, a_2, ..., a_n\}$ is an orthogonal basis for V. Let $w \in V$. Prove that $w = \frac{<w,a_1>}{<a_1,a_1>}a_1 + \cdots + \frac{<w,a_n>}{<a_n,a_n>}a_n$

$w \in V$ use Gram-Schmidt to get an element orthogonal to $\{a_1, a_2, -- a_n\}$ using w $\quad L = w - \frac{\langle w, a_1\rangle}{\langle a_1, a_1\rangle} a_1 - \frac{\langle w, a_2\rangle}{\langle a_2, a_2\rangle} a_2 - \cdots - \frac{\langle w, a_n\rangle}{\langle a_n, a_n\rangle} a_n$

since $\dim(V) = n \Rightarrow L = 0_V$ ✓

$\Rightarrow w = \frac{\langle w, a_1\rangle}{\langle a_1, a_1\rangle} a_1 + \cdots + \frac{\langle w, a_n\rangle}{\langle a_n, a_n\rangle} a_n$

b) Now use the normal dot product on R_3. Assume that $\{(1,1,0), (0,0,1), a_3\}$ is an orthogonal basis for R^3. Then we know that $(2,2,4) = c_1(1,1,0) + c_2(0,0,1) + c_3a_3$. Find the value of c_2.

$c_2 = \frac{(2,2,4)\cdot(0,0,1)}{(0,0,1)\cdot(0,0,1)} = \frac{4}{1} = 4$ ✓

+1/2

QUESTION 6. (short answer) Let $T : R^3 \to R^3$ be a linear transformation such that $2, a, 4$ are eigenvalues of T and $F : R^3 \to R^3$ be a linear transformation that is NOT bijective (and hence non-invertible), where $F(w) = T^2(w) + 2T(W) + W$, for every $w \in R^3$. Prove that T is bijective.

$C_T(\alpha)$ is a polynomial of degree 3, $C_T(\alpha) = (\alpha-2)(\alpha-a)(\alpha-4)$ & $a \in \mathbb{R}$

$\Rightarrow T$ is diagonalizable $\Rightarrow T$ similar to a diagonal $D \Rightarrow M = QDQ^{-1}$

where $Q = [E_2 \ E_a \ E_4]$

Q is invertible

M is s.M.R of T

For $\alpha = a$, $a^2 + 2a + 1$ is an eigenvalue of F

if $a = 0 \Rightarrow 1$ is an eigenvalue of F

So eigenvalues of F are $9, 25, 1 \Rightarrow F$ similar to a diagonal D_2 that is invertible $\Rightarrow F$ invertible (contradiction) $(|D_2| \neq 0)$

see opp. page

We conclude that $a \neq 0 \Rightarrow D$ is invertible $\Rightarrow M$ is invertible $\Rightarrow T$ is bijective

QUESTION 7. Let $B = \{(1, 0, -1), (0, 1, -1), (0, 0, 1)\}$ be an ordered basis for R^3 and $B' = \{(2, -1), (-3, 2)\}$ be an ordered basis for R^2. Let $T : R^3 \to R^2$ be a linear transformation over R such that $T(1, 0, -1) = (1, 0), T(0, 1, -1) = (1, 0), T(0, 0, 1) = (0, 1)$.

$b_1 \quad b_2 \quad b_3$

(i) Find the matrix representation of T with respect to B and B', i.e. $M_{B,B'}$.

$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$ $\quad Q^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ (opp. page) $\quad W = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Let M be matrix Rep of T

$M = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T(b_1)\ T(b_2)\ T(b_3)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$W^{-1} = \frac{1}{4-3} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$M_{B,B'} = W^{-1} M Q = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

(ii) Find a general formula for $[(a, b, c)]_B$. Then find $[(2, 4, 4)]_B$.

$[(a,b,c)]_B = Q^{-1} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$= a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = (a, b, a+b+c)$

$[(2,4,4)]_B = (2, 4, 10)$

2

10/10

Please see opposite page for parts of Q 6 & Q7

(iii) Find $[T(2,2,4)]_{B'}$. Now find $T(2,2,4)$.

$$[T(2,4,4)]_{B'} = M_{B,B'} \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix} = \begin{bmatrix} 4+8+30 \\ 2+4+20 \end{bmatrix}$$

$$= \begin{bmatrix} 42 \\ 26 \end{bmatrix} = (42, 26)$$

$$T(2,4,4) = 42\begin{pmatrix} 2 \\ -1 \end{pmatrix} + 26\begin{pmatrix} -3 \\ 2 \end{pmatrix} = (84-78, -42+52) = (6,10)$$

(iv) Find the standard matrix of T.

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(s.m.R of T)

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 2+4+0 \\ 2+4+4 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

(check)

(v) Write Range T as span of some basis

Range(T) = span$\{(1,1),(0,1)\}$

(vi) write Z(T) (ker(T)) as span of some basis

$$M \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x_3 = 0$

$x_1 + x_2 = 0$

$x_1 = -x_2$

Z(T) = span$\{(-1,1,0)\}$

$T: \mathbb{R}^4 \to \mathbb{R}^4$

QUESTION 8. (short proof) Consider the normal dot product on R^4. Let $T \in Hom_R[R^4, R^4]$ such that $T(w) = T^*(w)$ for every $w \in R^4$. Prove that the roots of $C_T(x)$ are all real numbers.

Let M be matrix rep. of T

Since $T(w) = T^*(w)$ $\Rightarrow$ $\underbrace{(\overline{M})^T}_{M^*} = M$ (Hermitian)

As per the fact in class notes

roots of $C_T(x)$ are all real numbers

QUESTION 9. Consider the Fake dot product on $R^{2\times 2}$. Let $W = span\{\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}, \begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\}$. Find the orthogonal complement of W.

Let $M = \begin{bmatrix} 1 & 0 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$ solve homogenous (M u.m.R of $T: \mathbb{R}^4 \to \mathbb{R}^2$)

$R_1 + R_2 \to R_2$ $\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix}$

$x_1 + x_3 + x_4 = 0 \Rightarrow x_1 = -x_3 - x_4$

$x_2 + 2x_3 + 2x_4 = 0 \Rightarrow x_2 = -2x_3 - 2x_4$

$Z(T) = \{(-x_3 - x_4, -2x_3 - 2x_4, x_3, x_4) \mid x_3, x_4 \in \mathbb{R}\}$

$= \{x_3(-1,-2,1,0), x_4(-1,-2,0,1) \mid x_3, x_4 \in \mathbb{R}\}$

$= span\{(-1,-2,1,0), (-1,-2,0,1)\} = Range(T^*)$

$W^\perp = span\{\begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}\}$ ← Translate to $\mathbb{R}^{2\times 2}$

QUESTION 10. (1) Let V be a normed vector space over R. Prove that $<x, y> = (||x+y||^2 - ||x-y||^2)/4$.

$||x+y||^2 = <x+y, x+y> = <x,x> + <y,y> + 2<x,y>$

$= ||x||^2 + ||y||^2 + 2<x,y>$ — (1)

$||x-y||^2 = <x-y, x-y> = <x,x> - <x,y> - <x,y> + <y,y>$

$= ||x||^2 + ||y||^2 - 2<x,y>$ — (2)

$||x+y||^2 - ||x-y||^2 = 4<x,y>$ By (1) & (2)

$\Rightarrow <x,y> = \dfrac{||x+y||^2 - ||x-y||^2}{4}$

(2) Consider the dot product on R^n. Give me an example of a linear transformation $T \in Hom_R(R^4, R^4)$ such that $< T(v), v >= 0$ for every $v \in R^4$, but $T(y) \neq 0$ for some $y \in R^4$. (i.e., T need not be the trivial linear transformation)

$T(1,0,0,0) = (0,1,0,0)$

$T(0,1,0,0) = (0,0,1,0)$

$T(0,0,1,0) = (0,0,0,1)$

$T(0,0,0,1) = (1,0,0,0)$

$T: \mathbb{R}^4 \to \mathbb{R}^4$

$M \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{I_4} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{K}$

$\Rightarrow M = K$ s.m.R of T

$T(a_1, a_2, a_3, a_4) = M \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$

No

$= \begin{bmatrix} 0 \\ a_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ a_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_3 \end{bmatrix} + \begin{bmatrix} a_4 \\ 0 \\ 0 \\ 0 \end{bmatrix} = (a_4, a_1, a_2, a_3)$

QUESTION 11. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$. Find the Smith form of A. i.e., Find invertible matrices R and C over Z such that $RAC = D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, where $|A| = \pm|D|$ and $a \mid b \mid c$.

gcd of $A = 1$

$a = \pm 1$

$|A| = 3\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 3(2-1) = 3$

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$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1 + R_2 \to R_2 \quad \sim$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C_1 + C_2 \to C_2 \quad \sim$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -C_1 + C_3 \to C_3 \quad \sim$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -2C_2 + C_3 \to C_3 \quad \sim$$

$$\underset{R}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}\underset{D}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}}\underset{C}{\begin{bmatrix} 1 & 1 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\underset{R}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}\underset{A}{\begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}}\underset{C}{\begin{bmatrix} 1 & 1 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = D$$

$a = 1 \quad b = 1 \quad c = 3$